

Creation, manipulation and detection of Majorana fermions with cold atoms in optical lattice

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We propose an experimental scheme to simulate the transverse field Ising model with cold atoms trapped in one-dimensional optical lattice. Majorana fermions are created at the ends of the optical lattice segment in topological phase. By controlling the addressing lasers, one can move, fuse and braid them. We also show that the non-Abelian braiding statistics of Majorana fermions can be demonstrated unambiguously through the construction of two braiding operations and distinguishing the resulting two output orthogonal collective spin states. A nice feature of the scheme is that the strong fluorescence provided by the collective spin state can be readily detected in experiment.

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Majorana fermions are particles that are their own antiparticles unlike Dirac fermions where electrons and positrons are distinct [1]. When exchanged among themselves, Majorana fermions obey non-Abelian statistics. Such fundamentally interesting particles have recently attracted much attention due to their potential application in topological quantum computation [2]. Majorana modes are originally perceived as zero-energy states bound to the vortices in two-dimensional (2D) spinless $p_x \pm ip_y$ -wave superconductor (SPSC) [3] or the two ends in one-dimensional (1D) SPSC chain [4]. It has been recently proposed that a semiconductor thin film with Rashba spin-orbit coupling, together with proximity-induced superconductivity and Zeeman splitting, resembling the SPSC model, can be used to create Majorana fermions [5]. One dimensional version of this system has been shown to host Majorana fermions at the two ends of a semiconducting wire [6]. By using a T-junction wire network, the Majorana fermions in the 1D semiconducting wires can be braided by tuning the local gates [7]. Despite the relative ease with which Majorana fermions can be stabilized in 1D wires making this 1D set-up more promising as a topological quantum information processing platform, experimental challenges need to be overcome: strong Zeeman fields could destroy the superconductor and the local control of electron density by the

gates is non-trivial due to strong screening by the superconductor.

Cold atoms nowadays are widely recognized as powerful experimental tools for mimicking a wide range of systems originally stemming from condensed-matter physics. This system can provide a highly controllable and tunable environment. Many significant experimental advances have been made in this forefront field. In particular, recent experiments have realized synthetic magnetic fields and spin-orbit coupling for ultracold atoms [8, 9]. Several protocols have been put forward to use these technologies for generating and probing Majorana fermions in 2D and 1D fermionic gases [10–15]. However, the braiding Majorana fermions as well as the detection of their non-Abelian statistics remains an outstanding problem.

In this work, we propose an experimental scheme to create and braid Majorana fermions as well as to detect their non-Abelian statistics with cold atoms in optical lattice. Note that the transverse field Ising model (TFIM) after Jordan-Wigner (JW) transformation is equivalent to the 1D SPSC model [16, 17]. We show that various TFIMs can be simulated with two-component ultracold bosonic atoms trapped in a 1D optical lattice. This atom-lattice system is highly tunable and very robust. The spin exchange coupling and the transverse field can be tuned by changing the laser intensity. Under this tuning, the optical lattice can be driven into topological phase or non-topological phase. Majorana fermions are bound to the ends of the optical lattice segment in topological phase. Such tuning can also allow Majorana fermions to be moved, recreated and fused. By further employing a cross lattice, Majorana fermions

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can be braided in the 1D optical lattice. In addition, the orientation of our simulated Ising coupling can be conveniently rotated by varying the phase of the laser. Finally, we show that such rotation can also mimic the braiding of Majorana fermions.

Unambiguous detection of non-Abelian statistics of Majorana fermions through easily measurable collective spin states is another distinct advantage of our proposal. We use the braid group elements of Majorana fermions to construct two different orders of braiding operations. When applied to the same initial state, the corresponding output states are orthogonal. For 1D superconductor model, the state of the non-local Majorana fermions has neutral charge and it is hard to be detected. However, the Majorana fermion states in our spin chain model can be mapped back to spin basis so that the states of two output Majorana fermions are two orthogonal collective spin states. Compared with the detection of the fluorescence from single atom [10, 11], which would be hard, these collective states consist of hundreds of atoms and they can provide strong fluorescence, which allows for experimental detection with high efficiency. In this way, our proposed scheme provides an easy way to detect the fundamental non-Abelian statistics of Majorana fermions.

Let us consider an ensemble of ultracold ^{87}Rb atoms trapped in a 1D optical lattice at 1064 nm. With standard optical pumping methods, we suppose that the atoms can only be in the states $|5S_{1/2}, F=2, m_F=1\rangle$ or $|5S_{1/2}, F=1, m_F=0\rangle$ denoted in the following text with an effective spin index $\sigma = \downarrow, \uparrow$. Taking into account the ground state hyperfine splitting $\Delta\nu_{hf} = 6.8$ GHz, the AC polarizabilities for the spin up and spin down have a small difference of about 3%. As shown in Fig. 1, it can be used to generate a state-independent optical lattice by applying a standing wave laser beam L_1 . By considering a trapping depth of $20E_r$, where E_r is the lattice recoil energy, the 1D potential is almost state-independent within a mismatch on the hopping rate lower than 15%. In order to simulate the Ising model in this 1D optical lattice, we need to apply a second state-dependent optical lattice with a potential V_σ . As we will see, this state-dependent potential is very important for realizing the Ising model.

At sufficiently low temperatures and strong lattice potential, the atoms are confined to the ground state of the optical lattice, and the system can be described by a Bose-Hubbard Hamiltonian

$$H = - \sum_{\langle i,j \rangle \sigma} (t_\sigma a_{i\sigma}^\dagger a_{j\sigma} + H.c) + \frac{1}{2} \sum_{i,\sigma\sigma'} U_{\sigma\sigma'} : n_{i\sigma} n_{i\sigma'} : , \quad (1)$$

where $a_{i\sigma}$ is the bosonic annihilation operator for atom states of spin σ at the lattice site i , $n_{i\sigma} = a_{i\sigma}^\dagger a_{i\sigma}$, $: (...):$ denotes normal order of the product of creation-annihilation operators, t_σ is the spin-dependent hopping rate and $U_{\sigma\sigma'}$ is the on-site energy between species σ and σ' . It is known that t_σ depends sensitively upon the lattice potential while $U_{\sigma\sigma'}$ exhibits weak dependence [18].

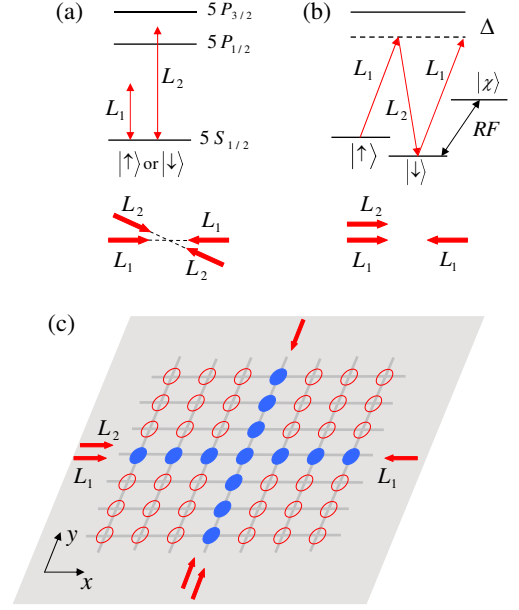


FIG. 1: (Color online) Energy diagram and laser beam configurations for the σ^z (a) and σ^ϕ (b) Ising models. L_1 and L_2 are the lasers leading to the 1D state-independent optical lattice and spin-dependent hopping rate. (c) 2D lattice for the braiding of Majorana fermion. The filled blue (open red) circles symbolized the atoms in the $|\sigma\rangle$ ($|\chi\rangle$) state. An RF-field is supposed to be used to flip the state in $|\chi\rangle$ to the spin state $|\sigma\rangle$.

Moreover, for the state under consideration the scattering lengths are $a_{\uparrow\uparrow} \simeq a_{\downarrow\downarrow} \simeq a_{\uparrow\downarrow}$. Thus, in what follows, $U_{\sigma\sigma'} = U$ is assumed. By varying the intensity of trapping laser to change the lattice potential, one can control the ratio U/t_σ to realize the Mott insulator with unitary filling [19], we will now consider this case in our model.

By utilizing an additional state-dependent potential, the above Hubbard Hamiltonian is reduced to the σ^z - and σ^ϕ -Ising models. For σ^z -Ising model, as sketched in Fig. 1(a), we apply a weak standing-wave laser beam L_2 , detuned between the D2 (780 nm) and D1 (794 nm) lines. In order to make complete overlap with the first state-independent optical lattice, the standing-wave laser beam should have a proper angle of incidence to the first one. If one tunes the intensity of the laser beam so that $V_\uparrow \gg V_\downarrow$ (or vice-versa), the hopping rate t_\uparrow becomes negligible while t_\downarrow remains finite, after compensating effective B-fields, the effective Hamiltonian of Eq. (1) could be reduced to the σ^z -Ising model $J_1 \sum_{j=1}^{N-1} \sigma_j^z \sigma_{j+1}^z$ with $J_1 = -t_\downarrow^2/2U$ [18]. This method requires an interferometric control of the spatial overlapping of the two standing waves at the atomic cloud position, which may be experimentally challenging.

An alternative and simpler method, for σ^ϕ -Ising model, consists of coupling the effective spin states via a two-photon Raman process. One photon is issued from the 1D lattice beams with a Rabi frequency $\Omega_1 \cos^2 kx$,

where k is the wave number of the laser. The second photon comes from a running wave, L_2 , propagating along $x > 0$ with a complex Rabi frequency $\Omega_2 e^{i\varphi}$ (see Fig. 1(b)). At the equilibrium position of the atoms, the complex Rabi frequency of the Raman coupling is $\Omega_R = \Omega_1 \Omega_2 e^{i\varphi} / \Delta$, where Δ is the one photon detuning. One notes that φ can be controlled by standard phase lock techniques and adjusted over a temporal delay of $(4\pi\Delta\nu_{hf})^{-1} \simeq 12$ ps. At resonance, the Raman coupling generates the state-dependent dressed potential $V_2 = V_+ |+\rangle\langle+| + V_- |-\rangle\langle-|$, where $|\pm\rangle = (|\uparrow\rangle \pm e^{i\varphi} |\downarrow\rangle) / \sqrt{2}$, $V_+ = -V_- = \hbar |\Omega_R| / 2$. The total state-dependent potential barriers now become $\tilde{V}_s = V_0 + V_s$ ($s = \pm$). By improving the intensity of laser beam L_2 to make $\tilde{V}_+ \gg \tilde{V}_-$, the hopping rate t_+ for the atom in the dressed state $|+\rangle$ becomes negligible. This can be achieved with moderate laser power. In this case, the effective Hamiltonian of Eq. (1) can be rewritten as the σ^φ -Ising model $J_2 \sum_{j=1}^{N-1} \sigma_j^\varphi \sigma_{j+1}^\varphi$, where $J_2 = -t_-^2 / 2U$ and $\sigma^\varphi = \cos \varphi \sigma^x + \sin \varphi \sigma^y$.

To realize the TFIM, we still need to generate the transverse field. For transverse field σ^z -Ising model, an addressing laser beam with Rabi frequency Ω_x is applied to the lattice sites, which can generate the transverse field term $h_x \sigma_j^x$. Combined with the Ising Hamiltonian, we realize the transverse field σ^z -Ising model

$$H_1 = J_1 \sum_{j=1}^{N-1} \sigma_j^z \sigma_{j+1}^z + h_x \sum_{j=1}^N \sigma_j^x, \quad (2)$$

where the transverse field $h_x = \hbar \Omega_x / 2$. For transverse field σ^φ -Ising model, the transverse magnetic field $h_z \sigma_j^z$ can be generated by applying an external magnetic field or by detuning the 2 photons Raman. We get a state-dependent AC shift $V_z \sigma_j^z$, which is equivalent to the effective transverse magnetic field. Thus we arrive at the transverse field σ^φ -Ising model

$$H_2 = J_2 \sum_{j=1}^{N-1} \sigma_j^\varphi \sigma_{j+1}^\varphi + h_z \sum_{j=1}^N \sigma_j^z, \quad (3)$$

where the transverse magnetic field $h_z = V_z$. In particular, when the laser phase φ is tuned to 0 or $\pi/2$, the above model is with respect to the transverse field σ^x - or σ^y -Ising model. As one can see, our simulated TFIM is highly tunable based on controlling the lasers, including tuning the orientations of Ising interaction, the spin exchange coupling and the transverse magnetic field.

Kitaev has shown that the transverse field σ^x -Ising model after a JW transformation is equivalent to the 1D SPSC model with superconducting phase $\theta = 0$ [16]. This result also holds true for transverse field σ^y - and σ^z -Ising models. Here we further show that our simulated transverse field σ^φ -Ising model can be reduced to 1D SPSC model with tunable superconducting phase.

For our purpose, through employing the string-like annihilation and creation operator $a_j = \sigma_j^- \prod_{i=1}^{j-1} \sigma_i^z$ and $a_j^+ = \sigma_j^+ \prod_{i=1}^{j-1} \sigma_i^z$, the transverse field σ^φ -Ising model can be rewritten as

$$H_2 = J_2 \sum_{j=1}^{N-1} (e^{i\varphi} a_j - e^{-i\varphi} a_j^+) (e^{i\varphi} a_{j+1} + e^{-i\varphi} a_{j+1}^+) + h_z \sum_{j=1}^N (2a_j^+ a_j - 1), \quad (4)$$

where the Dirac fermions satisfy the anticommutation relationship $\{a_i, a_j^+\} = \delta_{ij}$. By associating the spin exchange coupling J_2 to the hopping amplitude and the superconducting gap, the laser phase φ to the superconducting phase and the magnetic field h_z to the chemical potential, the transverse field σ^φ -Ising model is mapped into the 1D SPSC model with tunable superconducting phase. Based on the N Dirac fermions annihilation and creation operators, $2N$ Majorana fermions operators are defined as

$$\gamma_{A,j} = e^{i\varphi} a_j + e^{-i\varphi} a_j^+, \quad \gamma_{B,j} = i(e^{-i\varphi} a_j^+ - e^{i\varphi} a_j), \quad (5)$$

where $\gamma_{\alpha,j}^2 = 1$ and $\{\gamma_{\alpha,j}, \gamma_{\beta,k}^+\} = 2\delta_{\alpha\beta} \delta_{jk}$, $\alpha, \beta = A, B$. In terms of these operators, the Hamiltonian in Eq. (4) becomes

$$H_2 = iJ_2 \sum_{j=1}^{N-1} \gamma_{B,j} \gamma_{A,j+1} + ih_z \sum_{j=1}^N \gamma_{A,j} \gamma_{B,j}. \quad (6)$$

When $J_2 \neq 0$ and $h_z = 0$, the system is in the topological phase with two unpaired Majorana end modes $\gamma_{A,1}$ and $\gamma_{B,N}$, while for $J_2 = 0$ and $h_z \neq 0$, there is no Majorana end modes and the lattice is in the non-topological phase. In the previous case, the two Majorana fermions can be combined into an ordinary non-local Dirac fermion $c = (\gamma_{A,1} + i\gamma_{B,N})/2$, which yields two degenerate ground states $|0\rangle$ and $|1\rangle = c^+ |0\rangle$ [4]. Because the fermion parity $P = -i \prod_{j=1}^N \gamma_{A,j} \gamma_{B,j}$ has a Z_2 symmetry, the ground states $|0\rangle$ and $|1\rangle$ have even and odd parity, i.e. $P|0\rangle (|1\rangle) = |0\rangle (-|1\rangle)$. Due to the nonlocality of Majorana fermions that protects the ground states from decoherence, one can use them to encode a topological qubit for topological quantum memory.

The spin exchange coupling J_2 and the transverse field h_z can be tuned by controlling the addressing laser beams. As displayed in Fig. 2(a-b), such tuning allows Majorana fermions to be created, moved and fused in the 1D optical lattice. In Fig. 2(a), we assume that the lattice is a simulated transverse field σ^φ -Ising model and we can address its two edge sites with two laser beams, one for improving the local state-independent potential to make $J_2 = 0$ and one for generating the transverse field h_z . The two edge lattice segments are driven into

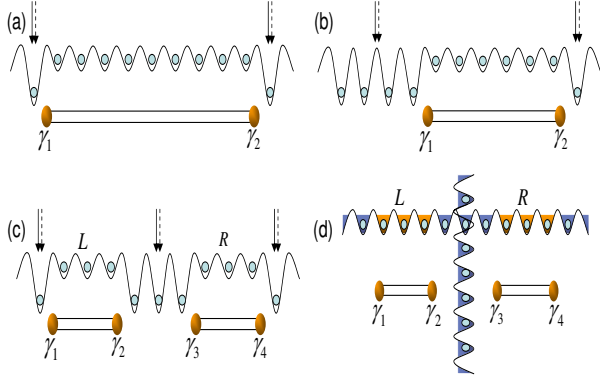


FIG. 2: (Color online) Schematic setup for manipulating and braiding Majorana fermions. By controlling the addressing lasers, one pair of Majorana fermions (γ_1, γ_2) is created and moved in (a) and (b). With the same control, two optical lattice segments L and R are driven into topological phase and two pairs of Majorana fermions (γ_1, γ_2) and (γ_3, γ_4) are generated in (c). To braid these Majorana fermions, a cross lattice is introduced in (d), where the orange segment denotes the topological phase while the blue one denotes non-topological.

non-topological phase but the center segment remains in the topological phase, which leads to one pair of Majorana fermions γ_1 and γ_2 created at the ends of the topological segments. In Fig. 2(b), by sequentially driving the leftmost lattice segment non-topological, the Majorana fermion γ_1 is moved rightward and finally fused when it meets the Majorana fermion γ_2 . The experimental requirement of single site addressing is now possible with high resolution microscope [20, 21]. As we will see later, this manipulation constitutes basic operations needed to exchange Majorana fermions and realize their non-Abelian braiding statistics.

Another feature of our model is that the superconducting phase for JW fermions is tunable. Interestingly, we find that this tuning can be used to simulate the braiding of the Majorana fermions at the ends of the same topological segment. As described in Fig. 2(a), two Majorana fermions $\gamma_{A,1}$ and $\gamma_{B,N}$ are located at the ends of the center topological lattice segment. According to Eq. (5), they can be written as $\gamma_{A,1} = e^{i\varphi}c + e^{-i\varphi}c^\dagger$, $\gamma_{B,N} = i(e^{-i\varphi}c^\dagger - e^{i\varphi}c)$. By adiabatically tuning the laser phase from φ to $\varphi \pm \pi/2$, we get the following transformation $\gamma_{A,1} \rightarrow \mp \gamma_{B,N}$, $\gamma_{B,N} \rightarrow \pm \gamma_{A,1}$, which just implements the braiding operation $\tau = \exp(\mp \frac{\pi}{4} \gamma_{B,N} \gamma_{A,1}) = \exp(\mp \frac{\pi}{4} \tilde{\sigma}_z)$ [7], where $\tilde{\sigma}_z$ is a Pauli matrix in the fermionic occupation basis $\{|0\rangle, c^\dagger|0\rangle\}$. Moreover, adiabatic change of the laser phase φ by π will result in a $\tilde{\sigma}_z$ quantum gate operation. These operations are expected to be used for manipulating the quantum information stored in the topological quantum memory.

We now consider the detection of the non-Abelian braiding statistics of Majorana fermions. It is known that, if one exchanges the Majorana fermion γ_i with

its nearest neighbor one γ_{i+1} , one can realize the braiding operation $\tau_i = \exp(\frac{\pi}{4} \gamma_{i+1} \gamma_i)$ [22]. The non-Abelian braiding statistics arises from the fact that $\tau_i \tau_{i+1} \neq \tau_{i+1} \tau_i$. So in order to demonstrate this non-Abelian braiding statistics, we at least need four Majorana fermions. As shown in Fig. 2(c), by addressing some particular optical lattice sites, the two optical lattice segments L and R are driven into topological phases and two pairs of Majorana fermions (γ_1, γ_2) and (γ_3, γ_4) are created at their ends. Based on these Majorana end modes, we define two Dirac fermions $c_L = (\gamma_1 + i\gamma_2)/2$ and $c_R = (\gamma_3 + i\gamma_4)/2$. In Fig. 2(d), as the way in the superconducting wire model [7], we introduce a cross lattice and use the basic operations in Fig. 2(a-b) to braid the Majorana fermions at the ends of the same or different topological segments, which correspond to the braiding operations $\tau_{1(3)}$ and τ_2 . Following the experimental method in [23], as shown in Fig. 1(c), the braiding operations involving two orthogonal optical lattice segments can be realized by using a 2D Mott insulator with unitary filling. Initially the atoms in the 2D Mott state are in a spin state $|\chi\rangle \neq |\sigma\rangle$. Using single spin addressing, the two orthogonal lattice segments are created in the $|\sigma\rangle$. If $V_{\chi,\sigma} \gg V_{\sigma,\sigma'}$, the dynamic is frozen between $|\chi\rangle$ and $|\sigma\rangle$ neighboring sites. Here we assume the lattice has simulated the transverse field σ^x -Ising model. In the fermionic occupation basis $\{|00\rangle_{LR}, c_L^\dagger|00\rangle_{LR}, c_R^\dagger|00\rangle_{LR}, c_L^\dagger c_R^\dagger|00\rangle_{LR}\}$, the two braiding operations τ_1 and τ_2 are represented by

$$\tau_1 = \frac{1}{\sqrt{2}} \text{diag}(1-i, 1+i, 1-i, 1+i), \quad (7)$$

$$\tau_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & -i \\ & 1 & -i & \\ & -i & 1 & \\ -i & & & 1 \end{pmatrix}. \quad (8)$$

Based on these operations, we construct two composite braiding operations to demonstrate $\tau_1 \tau_2 \neq \tau_2 \tau_1$. The two composite braiding operations are chosen as $S = \tau_1 \tau_2$, $T = \tau_2 \tau_1$ with the properties

$$ST = -i\tilde{\sigma}_x^L \otimes \tilde{\sigma}_x^R, \quad TS = -i\tilde{\sigma}_z^L \otimes I^R. \quad (9)$$

Suppose the two Majorana fermions are initially prepared in the state $(|0\rangle + |1\rangle)_L \otimes (|0\rangle + |1\rangle)_R / 2$, after the braiding operations ST and TS , the output states $-i(|0\rangle + |1\rangle)_L \otimes (|0\rangle + |1\rangle)_R / 2$ and $-i(|0\rangle - |1\rangle)_L \otimes (|0\rangle + |1\rangle)_R / 2$ are orthogonal with each other. Thus we can demonstrate the non-Abelian nature of the Majorana fermions unambiguously by detecting the difference of the two orthogonal output states.

In fact, this detection is readily available in experiment by transferring them into spin basis. In this basis, the two degenerate ground states of the topological lattice segment i are written as

$$|0\rangle_i = \frac{|+\rangle^{\otimes m_i} + |-\rangle^{\otimes m_i}}{\sqrt{2}}, \quad |1\rangle_i = \frac{|+\rangle^{\otimes m_i} - |-\rangle^{\otimes m_i}}{\sqrt{2}}, \quad (10)$$

where $|\pm\rangle = (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$. The Dirac fermion and the fermion parity operators become $c_i = (\sigma_1^x - \sigma_{m_i}^x \prod_{j=1}^{m_i} \sigma_j^z)/2$ and $P_i = \prod_{j=1}^{m_i} \sigma_j^z$, where $i = L, R$, m_i is the total lattice sites in the topological segment i . Using these expressions, one can demonstrate $c_i^+ |0\rangle_i = |1\rangle_i$, $c_i |1\rangle_i = |0\rangle_i$ and $P_i |0\rangle_i (|1\rangle_i) = |0\rangle_i (-|1\rangle_i)$ as in the JW fermions basis. By substituting Eq. (10), the initial state of Majorana fermions is transformed into $|+\rangle^{\otimes m_L} |+\rangle^{\otimes m_R}$, which can be easily prepared by optical pumping, and the two output states become $-i |+\rangle^{\otimes m_L} |+\rangle^{\otimes m_R}$ and $i |-\rangle^{\otimes m_L} |+\rangle^{\otimes m_R}$, which can be adiabatically rotated into $-i |\uparrow\rangle^{\otimes m_L} |\uparrow\rangle^{\otimes m_R}$ and $i |\downarrow\rangle^{\otimes m_L} |\uparrow\rangle^{\otimes m_R}$ by another laser driving. That is, we only need to distinguish the two orthogonal collective spin states $|\downarrow\rangle^{\otimes m_L}$ and $|\uparrow\rangle^{\otimes m_L}$. For this purpose, one can apply one laser beam on the lattice segment L to couple the spin up state to an auxiliary excited state and detect the fluorescence of its emissions. The cases with and without fluorescence are corresponding to the state $|\uparrow\rangle$ and $|\downarrow\rangle$. The distinct advantage here is that the state $|\uparrow\rangle^{\otimes m_L}$ is a collective spin state which can provide a strong fluorescence to distinguish itself from the state

$|\downarrow\rangle^{\otimes m_L}$. Such property makes our proposal much more appealing for the detection the non-Abelian statistics of Majorana fermions.

In summary, we have proposed an experimental scheme to create and manipulate Majorana fermions. In the scheme, the transverse field Ising model has been simulated using cold atoms trapped in optical lattice. By tuning the intensity of the addressing lasers, Majorana fermions can be generated at the ends of the topological lattice segment. Such tuning can also allow them to be moved, fused and braided. Finally, we have shown how to detect the non-Abelian statistics of Majorana fermions by distinguishing two orthogonal output collective spin states immediately after performing two opposite orderings of braiding operations.

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